

Spin-electron acoustic waves: The Landau damping and ion contribution in the spectrum

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Separated spin-up and spin-down quantum kinetics is derived for more detailed research of the spin-electron acoustic waves. Kinetic theory allows to obtain spectrum of the spin-electron acoustic waves including effects of occupation of quantum states more accurately than quantum hydrodynamics. We apply quantum kinetic to calculate the Landau damping of the spin-electron acoustic waves. We have considered contribution of ions dynamics in the spin-electron acoustic wave spectrum. We obtain contribution of ions in the Landau damping in temperature regime of classic ions. Kinetic analysis for ion-acoustic, zero sound, and Langmuir waves at separated spin-up and spin-down electron dynamics is presented as well.

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I. INTRODUCTION

Recently developed separate spin evolution quantum hydrodynamic (SSE-QHD) model [1], giving separated description of spin-up and spin-down electrons, allowed us to discover new type of longitudinal collective excitations in degenerate quantum plasmas. This excitation is called the spin-electron acoustic wave (SEAW). In this model, spin-up and spin-down electrons are considered as two different species. The SEAW exists in magnetised plasmas due to difference of the Fermi momentum of spin-up and spin-down electrons at presence of an external magnetic field.

Propagation of the SEAWs parallel and perpendicular to an external magnetic field was considered in Ref. [1]. Further research of the SEAWs was performed in Refs. [2] and [3]. Dispersion of the SEAWs in different two dimensional structures was studied in Ref. [3]. Plane-like two dimensional electron gas in a magnetic field perpendicular to the sample, and conducting nanotubes, having cylindrical geometry, in an external magnetic field parallel to the cylinder axis were considered in Ref. [3].

More general case of oblique wave propagation in three dimensional structures was considered in Ref. [2]. It was demonstrated that at oblique propagation we have two branches of the SEAWs instead of one existing in limit cases of parallel and perpendicular propagation.

In paper [1], derivation of the separated spin evolution QHD with two different species of spin-up and spin-down electrons was demonstrated on a simple example of the single-particle Pauli equation. It is rather obvious that a single-particle equation has nothing to do with a plasma description. A full derivation should be based upon a many-particle theory, as, for instance, the many-particle QHD (MPQHD) developed by Kuz'menkov and

coauthors [4], [5], [6], [7], [8], [9]. The final equations, presented in Ref. [1], were obtained by the corresponding modification of the MPQHD. Hence they have more general form than the result of the separate spin-up and spin-down fluidisation of the single-particle Pauli equation. Nevertheless application of the single-particle Pauli equation was a simple way to demonstrate the correct structure of the SSE-QHD. This is a generalisation of famous fluidisation of the single-particle Pauli equation performed by Takabayasi [10].

Consequences of separate spin evolution for the Langmuir [1], [2], [3] and Trivelpiece-Gould [2] waves were also studied in mentioned papers.

This paper is dedicated to further analysis of the spin-electron acoustic waves and influence of separate spin evolution of electrons on ion acoustic and zeroth sound waves. In this paper we focus our attention on waves propagating parallel to the external magnetic field. Here we develop the separate spin evolution quantum kinetics. Kinetic theory gives a background for more careful analysis of distribution of electrons over different quantum states and contribution of these effects in the quantum plasma properties.

Since we have an example of derivation of SSE-QHD from the single particle Pauli equation, we stress attention on a many-particle derivation of separate spin evolution quantum kinetics.

The separate spin evolution quantum kinetics allows to calculate the Landau damping of the SEAWs. We apply the separate spin evolution quantum kinetics to obtain the Landau damping of the SEAWs and other excitations in two different regimes: regime of intermediate temperatures, when electrons are degenerate and ions are classical, and regime of low temperatures when all species are degenerate.

Some topics in quantum plasmas were discussed in reviews [11], [12], [13].

This paper is organized as follows. In Sec. II we describe basic definitions of quantum kinetics and describe

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quantum mechanic background essential for derivation of the quantum kinetics. Sec. III contains closed set of separate spin evolution quantum kinetic equations. In Sec. IV equilibrium state is described. Linearised kinetic equations for small perturbations of the equilibrium are presented in Sec. IV as well. In Sec. V a general form of dispersion equation for oblique propagating longitudinal waves is obtained. In Sec. VI we present detailed analysis of spectrum of longitudinal waves propagating parallel to the external magnetic field. In Sec. VII a brief summary of obtained results is presented.

II. METHOD OF DERIVATION OF SEPARATED SPIN-UP AND SPIN-DOWN QUANTUM KINETICS

A. Structure of many-particle N-spinor wave function

If we have a single particle with no spin degree of freedom it can be described by the wave function $\psi(\mathbf{r}, t)$, which is a complex function of three space coordinates and time. If we have two particles of that kind we need to apply the two-particle wave function $\psi(\mathbf{r}_1, \mathbf{r}_2, t)$, which is a complex function in six dimensional configuration space. It is hard to make assumptions for this function when we consider two interacting particles. However if interaction is weak, or we have two non-interacting particles, we can represent a two-particle wave function as the product of single particle wave functions $\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \psi(\mathbf{r}_1, t)\psi(\mathbf{r}_2, t)$, or including antisymmetry of fermion wave function $\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \frac{1}{2} \begin{vmatrix} \psi_{a_1}(\mathbf{r}_1, t) & \psi_{a_1}(\mathbf{r}_2, t) \\ \psi_{a_2}(\mathbf{r}_1, t) & \psi_{a_2}(\mathbf{r}_2, t) \end{vmatrix}$.

Next focus our attention on the spin-1/2 particles, as electrons, which are main subject of this paper. A single spin-1/2 particle is described by the spinor (the first-rank spinor) wave function, which is a two component wave function $\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$ (see for instance [14] section 56). Next step on the way of construction of the many-particle theory is obtaining of the wave function for two spin-1/2 particles. This wave function appears to be a second rank spinor [14].

Second-rank spinor is a four component quantity $\psi_{\zeta\tau}$ (see for instance [14] section 56 after formula 56.13). Components of $\psi_{\zeta\tau}$ are transformed as products $\psi_\zeta\psi_\tau$ of components of two first-rank spinors. The 2×2 unit matrix \hat{I} together with three sigma (Pauli) matrixes form a basis in space of the second-rank spinors.

Summarising all written above we can present a scheme of generalisation:

$$\left(\begin{array}{ccc} \psi(\mathbf{r}, t) & \Rightarrow & \psi(\mathbf{r}_1, \mathbf{r}_2, t) \\ \downarrow & & \downarrow \\ \begin{pmatrix} \psi_u(\mathbf{r}, t) \\ \psi_d(\mathbf{r}, t) \end{pmatrix} & \Rightarrow & \Psi(\mathbf{r}_1, \mathbf{r}_2, t) \end{array} \right) \quad (1)$$

where

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = \begin{pmatrix} \psi_{u1}(\mathbf{r}_1, \mathbf{r}_2, t) & \psi_{u2}(\mathbf{r}_1, \mathbf{r}_2, t) \\ \psi_{d1}(\mathbf{r}_1, \mathbf{r}_2, t) & \psi_{d2}(\mathbf{r}_1, \mathbf{r}_2, t) \end{pmatrix} \quad (2)$$

is the second rank spinor having presentation of 2×2 matrix. The two-particle wave function of spin-1/2 particles, being a matrix, should wear spinor subindexes, hence we write $\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = \Psi_{s_1 s_2}(\mathbf{r}_1, \mathbf{r}_2, t)$.

Spin and coordinate parts of the many-particle wave function can be separated in absence of the spin-current and spin-orbit interactions

$$\Psi(R, t) = \Psi_S(R, t) = \Lambda(R, t) \cdot \chi_S, \quad (3)$$

where $R = \{\mathbf{r}_1, \dots, \mathbf{r}_i, \dots, \mathbf{r}_N\}$ is the set of coordinates of N particles, $S = \{s_1, \dots, s_i, \dots, s_N\}$ is the set of spinor subindexes of N particles. Full many-particle wave function $\Psi(R, t)$ is antisymmetric to permutation of arguments. Hence if orbital part $\Lambda(R, t)$ is antisymmetric, then the spin part χ_S is symmetric. In opposite case orbital part $\Lambda(R, t)$ is symmetric, then the spin part χ_S is antisymmetric.

B. Many-particle Pauli equation as the starting point of derivation of kinetic equations

Thus we start with the many-particle Pauli equation

$$i\hbar\partial_t\Psi(R, t) = \left(\sum_{i=1}^N \left(\frac{1}{2m_i} \hat{\mathbf{D}}_i^2 + q_i\varphi_i^{ext} - \gamma_i \boldsymbol{\sigma}_i \mathbf{B}_{i(ext)} \right) + \frac{1}{2} \sum_{i,j \neq i}^N (q_i q_j G_{ij} - \gamma_i \gamma_j G_{ij}^{\alpha\beta} \sigma_i^\alpha \sigma_j^\beta) \right) \Psi(R, t), \quad (4)$$

where $\Psi(R, t) = \Psi_S(R, t)$, we can also rewrite terms containing spin operators with detail description of spinor indexes $\boldsymbol{\sigma}_i \Psi(R, t) = (\boldsymbol{\sigma}_i \Psi(R, t))_S = (\boldsymbol{\sigma}_i \Psi(R, t))_{s_1, \dots, s_i, \dots, s_N} = \sigma_{s_i s_{i'}}^\alpha \Psi_{s_1, \dots, s_{i'}, \dots, s_N}(R, t)$, and $\sigma_i^\alpha \sigma_j^\beta \Psi(R, t) = (\sigma_i^\alpha \sigma_j^\beta \Psi(R, t))_S = (\sigma_i^\alpha \sigma_j^\beta \Psi(R, t))_{s_1, \dots, s_i, \dots, s_j, \dots, s_N} = \sigma_{s_i s_{i'}}^\alpha \sigma_{s_j s_{j'}}^\beta \Psi_{s_1, \dots, s_{i'}, \dots, s_{j'}, \dots, s_N}(R, t)$.

Equation (4) governs evolution of N-spinor wave function $\Psi(R, t)$. In equation (4) m_i is the mass of particle with number i , below we consider system of particles with equal masses, q_i is the charge of particle, γ_i is the gyromagnetic ratio, for electrons it can be written as $\gamma_i \approx 1.00116\mu_B$, $\mu_B = q_i \hbar / (2m_i c)$ is the Bohr magneton, the difference of $|\gamma_e|$ from the Bohr magneton includes contribution of the anomalous magnetic dipole moment, φ_i^{ext} is the scalar potential of an external electromagnetic field acting on particle with number i , $\mathbf{B}_{i(ext)}$ is the external magnetic field, $(\hat{\mathbf{D}}_i \psi)(R, t) = ((-i\hbar \nabla_i - \frac{q_i}{c} \mathbf{A}_{i,ext})\psi)(R, t)$, with $\mathbf{A}_{i,ext}$ is the vector potential of an external electromagnetic field acting on

particle, σ_i is the Pauli matrixes describing spin of particles, \hbar is the reduced Planck constant, c is the speed of light, $\mathbf{B}_{i(ext)} = \nabla_i \times \mathbf{A}_{i(ext)}$, $\mathbf{E}_{i(ext)} = -\nabla_i \varphi_{ext}(\mathbf{r}_i, t) - \frac{1}{c} \partial_t \mathbf{A}_{ext}(\mathbf{r}_i, t)$, $G_{pn} = \frac{1}{r_{ij}}$ is the Green function of the Coulomb interaction containing module of the interparticle distance $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, and

$$G_{ij}^{\alpha\beta} = 4\pi\delta_{\alpha\beta}\delta(\mathbf{r}_{ij}) + \nabla_i^\alpha \nabla_i^\beta \frac{1}{r_{ij}} \quad (5)$$

is the Green function of the spin-spin interaction, δ_{ij} is the Kroneckers delta.

Let us present the explicit form of the Pauli matrixes

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (6)$$

The commutation relation for spin-1/2 matrixes is

$$[\hat{\sigma}^\alpha, \hat{\sigma}^\beta] = 2i\varepsilon^{\alpha\beta\gamma}\hat{\sigma}^\gamma, \quad (7)$$

with $\varepsilon^{\alpha\beta\gamma}$ is the Levi-Civita symbol.

C. Explicit form of the Pauli equation for two spin-1/2 particles

As the first simple example let us rewrite the Pauli equation (4) for a single particle in more explicit form

$$\begin{aligned} i\hbar\partial_t\psi_\uparrow = & \left(\frac{(\frac{\hbar}{i}\nabla - \frac{q_e}{c}\mathbf{A})^2}{2m} + q_e\varphi - \gamma_e B_z \right) \psi_\uparrow \\ & - \gamma_e(B_x - iB_y)\psi_\downarrow, \end{aligned} \quad (8)$$

and

$$\begin{aligned} i\hbar\partial_t\psi_\downarrow = & \left(\frac{(\frac{\hbar}{i}\nabla - \frac{q_e}{c}\mathbf{A})^2}{2m} + q_e\varphi + \gamma_e B_z \right) \psi_\downarrow \\ & - \gamma_e(B_x + iB_y)\psi_\uparrow. \end{aligned} \quad (9)$$

The single particle wave spinor can be easily presented as a linear combination of the spin-up and spin-down states described corresponding unit spinors

$$\psi_s(\mathbf{r}, t) = \psi_\uparrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \psi_\downarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (10)$$

It have allowed us to rewrite the Pauli equation in a rather more explicit form given be formulae (8) and (9).

Applying wave functions describing spin-up ψ_\uparrow and spin-down ψ_\downarrow states we can write probability density to find the particle in a point \mathbf{r} with spin-up $\rho_\uparrow = |\psi_\uparrow|^2$ or spin-down $\rho_\downarrow = |\psi_\downarrow|^2$. We also see $\rho = \rho_\uparrow + \rho_\downarrow$. Directions up \uparrow (down \downarrow) corresponds to spins having same (opposite) direction as (to) the external magnetic field. While magnetic moments have opposite to spin directions since we consider negatively charged particles.

Let us consider structure of the many-particle wave function $\Psi(R, t)$ in more details. To our end we will need different basis in space of the second-rank spinors:

$$\tau_1 = \frac{1}{2}(\hat{\sigma}_0 + \hat{\sigma}_z) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (11)$$

$$\tau_2 = \frac{1}{2}(\hat{\sigma}_x + i\hat{\sigma}_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (12)$$

$$\tau_3 = \frac{1}{2}(\hat{\sigma}_x - i\hat{\sigma}_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad (13)$$

and

$$\tau_4 = \frac{1}{2}(\hat{\sigma}_0 - \hat{\sigma}_z) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (14)$$

where

$$\hat{\sigma}_0 = \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (15)$$

is the unit second rank spinor.

It is essential to repeat that the two-particle wave function of two spin-1/2 particles is a 2×2 matrix (see formula (2)). Consequently we can expand it as a superposition of matrixes $\hat{\sigma}_0$ and $\sigma = \{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ or the set of matrixes $\{\hat{\tau}_a\}$ with $a = 1, 2, 3, 4$. So, the two-particle wave function has form of $\Psi = \sum_a \psi_a \hat{\tau}_a$, where $\psi_a = \psi_a(\mathbf{r}_1, \mathbf{r}_2, t)$ are complex functions.

Wave function of two spin-1/2 particles $\Psi_S(R, t)$ can be presented via the upper $\Psi_\uparrow(R, t)$, or lower $\Psi_\downarrow(R, t)$ line in the 2-rank spinor $\Psi_S(\mathbf{r}_1, \mathbf{r}_2, t) = \begin{pmatrix} \Psi_\uparrow(\mathbf{r}_1, \mathbf{r}_2, t) \\ \Psi_\downarrow(\mathbf{r}_1, \mathbf{r}_2, t) \end{pmatrix}$, where $\Psi_\uparrow(\mathbf{r}_1, \mathbf{r}_2, t) = (\psi_1(\mathbf{r}_1, \mathbf{r}_2, t) \ \psi_2(\mathbf{r}_1, \mathbf{r}_2, t))$, and $\Psi_\downarrow(\mathbf{r}_1, \mathbf{r}_2, t) = (\psi_3(\mathbf{r}_1, \mathbf{r}_2, t) \ \psi_4(\mathbf{r}_1, \mathbf{r}_2, t))$.

The density probability in the six dimensional configurational space appears in the traditional form $\rho(\mathbf{r}_1, \mathbf{r}_2, t) = \Psi^\dagger(R, t)\Psi(R, t)$. Its explicit form is $\rho(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_i \psi_i^* \psi_i$. We can separate terms in this sum in two groups $\rho_\uparrow(\mathbf{r}_1, \mathbf{r}_2, t)$ and $\rho_\downarrow(\mathbf{r}_1, \mathbf{r}_2, t)$ in the following way. We can introduce a notation $\rho_\uparrow(\mathbf{r}_1, \mathbf{r}_2, t) = \Psi_\uparrow^\dagger \Psi_\uparrow$, $\rho_\downarrow(\mathbf{r}_1, \mathbf{r}_2, t) = \Psi_\downarrow^\dagger \Psi_\downarrow$, which is not a mathematical symbol, but it will be very useful to get a compact form of formulae below. Applying wave functions describing spin-up Ψ_\uparrow and spin-down Ψ_\downarrow states of two particle systems we can write probability density to find both particles in point \mathbf{r}_1 and \mathbf{r}_2 with spin-up $\rho_\uparrow(\mathbf{r}_1, \mathbf{r}_2, t) = |\psi_1|^2 + |\psi_2|^2$ or spin-down $\rho_\downarrow = |\psi_3|^2 + |\psi_4|^2$. We also see $\rho = \rho_\uparrow + \rho_\downarrow$. Directions up \uparrow (down \downarrow) corresponds to spins having same (opposite) direction as (to) the external magnetic field.

A compact form of the Pauli equation for two spin-1/2 interacting particles can be written easily using the general form of the Pauli equation for N particles (4)

$$i\hbar\partial_t\Psi(R, t) = \left[\sum_{i=1}^2 \left(\frac{1}{2m_i} \hat{\mathbf{D}}_i^2 + q_i \varphi_i^{ext} - \gamma_i \sigma_i \mathbf{B}_{i(ext)} \right) \right] \Psi(R, t)$$

$$+ q_1 q_2 G_{12} - \gamma_1 \gamma_2 G_{12}^{\alpha\beta} \sigma_1^\alpha \sigma_2^\beta \Big] \Psi(R, t), \quad (16)$$

where $\Psi(R, t) = \Psi_{s_1, s_2}(\mathbf{r}_1, \mathbf{r}_2, t)$.

We are going to present a more explicit form of equation (16). To this end we introduce operator $\hat{\Lambda}$ as

$$\hat{\Lambda} = i\hbar\partial_t - \sum_{i=1}^2 \left(\frac{1}{2m_i} \hat{\mathbf{D}}_i^2 + q_i \varphi_i^{ext} \right) - q_1 q_2 G_{12}. \quad (17)$$

Finally we able to present explicit form of the Pauli equation for two particles involved in the Coulomb and spin-spin interactions, and also interacting with the external electromagnetic field

$$\begin{aligned} & \hat{\tau}_1 \left\{ \Lambda \psi_1 + \gamma_1 [B_{1z} \psi_1 + (B_{1x} - iB_{1y}) \psi_3] + \gamma_1 \gamma_2 [(G_{zx} + G_{zy} + G_{zz}) \psi_1 + (G_{xx} + G_{xy} + G_{xz}) \psi_3 - i(G_{yx} + G_{yy} + G_{yz}) \psi_3] \right\} \\ & + \hat{\tau}_2 \left\{ \Lambda \psi_2 + \gamma_2 [B_{2z} \psi_2 + (B_{2x} - iB_{2y}) \psi_4] + \gamma_1 \gamma_2 [(G_{xz} + G_{yz} + G_{zz}) \psi_2 + (G_{xx} + G_{yx} + G_{zx}) \psi_4 - i(G_{xy} + G_{yy} + G_{zy}) \psi_4] \right\} \\ & + \hat{\tau}_3 \left\{ \Lambda \psi_3 + \gamma_1 [-B_{1z} \psi_3 + (B_{1x} + iB_{1y}) \psi_1] + \gamma_1 \gamma_2 [-(G_{zx} + G_{zy} + G_{zz}) \psi_3 + (G_{xx} + G_{xy} + G_{xz}) \psi_1 + i(G_{yx} + G_{yy} + G_{yz}) \psi_1] \right\} \\ & + \hat{\tau}_4 \left\{ \Lambda \psi_4 + \gamma_2 [-B_{2z} \psi_4 + (B_{2x} + iB_{2y}) \psi_2] + \gamma_1 \gamma_2 [-(G_{xz} + G_{yz} + G_{zz}) \psi_4 + (G_{xx} + G_{yx} + G_{zx}) \psi_2 + i(G_{xy} + G_{yy} + G_{zy}) \psi_2] \right\} = 0. \end{aligned} \quad (18)$$

Comparing formula (18) with its analog for a single particle presented by equations (8) and (9) we see that two particle system is rather more complicate. It is essential to mention that two particle Pauli equation reflects many properties of N particle Pauli equation. Hence it allows to understand correct structure of quantum kinetics of electrons with separated spin-up and spin down evolution.

D. Basic definitions of quantum kinetics

Most famous quantum distribution function was suggested by Wigner [15], however we do not apply it here. We construct our kinetic theory in according with the many-particle quantum hydrodynamic (MPQHD) method [4], [6], [9]. We start with classic microscopic distribution function [16], [17]. We change classic dynamic functions of position and momentum of particles on the corresponding operators. As the result we find the operator of many-particle microscopic quantum distribution function [18], [19]

$$\hat{f} = \sum_i \delta(\mathbf{r} - \hat{\mathbf{r}}_i) \delta(\mathbf{p} - \hat{\mathbf{p}}_i). \quad (19)$$

Quantum mechanical averaging of the operator of many-particle distribution function gives us the microscopic distribution function for system of spinning parti-

cles [18], [19]

$$\begin{aligned} f_a(\mathbf{r}, \mathbf{p}, t) = \frac{1}{4} \int \left(\Psi^+(R, t) \sum_i \left(\delta(\mathbf{r} - \mathbf{r}_i) \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \right. \right. \\ \left. \left. + \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \delta(\mathbf{r} - \mathbf{r}_i) \right) \Psi(R, t) + h.c. \right) dR, \end{aligned} \quad (20)$$

for each species of particles $a = e$ for electrons and $a = i$ for ions. In formula (20) we have $\Psi^+(R, t) = \Psi_S^+(R, t) = (\Psi^S(R, t))^*$.

We can introduce the distribution function of subspecies of electrons for spin-up and spin-down electrons:

$$\begin{aligned} f_{e,s} = f_{e,s}(\mathbf{r}, \mathbf{p}, t) = \frac{1}{4} \int \left(\Psi_s^+(R, t) \sum_i \left(\delta(\mathbf{r} - \mathbf{r}_i) \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \right. \right. \\ \left. \left. + \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \delta(\mathbf{r} - \mathbf{r}_i) \right) \Psi_s(R, t) + h.c. \right) dR, \end{aligned} \quad (21)$$

for each subspecies of electrons.

In formula (21) we have applied functions $\Psi_S(R, t)$, which are the upper $\Psi_\uparrow(R, t)$, or lower $\Psi_\downarrow(R, t)$ line in the N-rank spinor $\Psi_S(R, t) = \begin{pmatrix} \Psi_\uparrow(R, t) \\ \Psi_\downarrow(R, t) \end{pmatrix}$.

III. SET OF KINETIC EQUATIONS

We consider quantum plasmas as the set of three species of particles: spin-up electrons, spin-down electrons and ions. Hence we have three kinetic equations presented below.

Kinetic equation for spin-up electrons is

$$\begin{aligned} \partial_t f_{e\uparrow} + \mathbf{v} \nabla_{\mathbf{r}} f_{e\uparrow} + q_e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{B}] \right) \nabla_{\mathbf{p}} f_{e\uparrow} \\ + \gamma_e \nabla B^z \cdot \nabla_{\mathbf{p}} f_{e\uparrow} + \frac{\gamma_e}{2} \left(\nabla B^x \cdot \nabla_{\mathbf{p}} S_{e,x} \right. \\ \left. + \nabla B^y \cdot \nabla_{\mathbf{p}} S_{e,y} \right) = \frac{\gamma_a}{\hbar} [S_{e,x} B_y - S_{e,y} B_x]. \end{aligned} \quad (22)$$

Kinetic equation for spin-down electrons has same structure as equation for spin-up electrons, but it has some different coefficients

$$\begin{aligned} \partial_t f_{e\downarrow} + \mathbf{v} \nabla_{\mathbf{r}} f_{e\downarrow} + q_e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{B}] \right) \nabla_{\mathbf{p}} f_{e\downarrow} \\ - \gamma_e \nabla B^z \cdot \nabla_{\mathbf{p}} f_{e\downarrow} + \frac{\gamma_e}{2} \left(\nabla B^x \cdot \nabla_{\mathbf{p}} S_{e,x} \right. \\ \left. + \nabla B^y \cdot \nabla_{\mathbf{p}} S_{e,y} \right) = -\frac{\gamma_a}{\hbar} [S_{e,x} B_y - S_{e,y} B_x]. \end{aligned} \quad (23)$$

Kinetic equation for ions is

$$\partial_t f_i + \mathbf{v} \nabla_{\mathbf{r}} f_i + q_i \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{B}] \right) \nabla_{\mathbf{p}} f_i = 0, \quad (24)$$

where we consider the charge-charge interaction only.

Considering the charge-charge and the spin-spin interactions we apply the self-consistent field approximation [13], [20], [17]. The MPQHD equations beyond the self-consistent field approximation can be found in Refs. [5], [7], [21].

Kinetic equations for electrons contain spin-distribution functions $S_{e,x}(\mathbf{r}, \mathbf{p}, t)$ and $S_{e,y}(\mathbf{r}, \mathbf{p}, t)$.

The spin distribution functions for each species appears as the quantum mechanical average of the corresponding operator

$$\hat{S}^\alpha = \sum_i \delta(\mathbf{r} - \hat{\mathbf{r}}_i) \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \hat{\sigma}_i^\alpha. \quad (25)$$

Hence explicit form of the spin distribution function is

$$\begin{aligned} S_a^\alpha(\mathbf{r}, \mathbf{p}, t) = \frac{1}{4} \int \left(\Psi_S^+(R, t) \sum_i \left(\delta(\mathbf{r} - \mathbf{r}_i) \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \right. \right. \\ \left. \left. + \delta(\mathbf{p} - \hat{\mathbf{p}}_i) \delta(\mathbf{r} - \mathbf{r}_i) \right) \hat{\sigma}_i^\alpha \Psi_S(R, t) + h.c. \right) dR. \end{aligned} \quad (26)$$

The spin distribution functions $S_{e,x}(\mathbf{r}, \mathbf{p}, t)$ and $S_{e,y}(\mathbf{r}, \mathbf{p}, t)$ appear for all electrons inspite the separation of spin-up and spin-down electrons in the distribution functions.

Differentiating explicit forms of S_x and S_y and applying the Pauli equation (8) and (9) for the time derivatives of the N-rank spinor wave function Ψ_S we obtain the following equations for spin-distribution functions of electrons

$$\begin{aligned} \partial_t S_{e,x} + \mathbf{v} \nabla_{\mathbf{r}} S_{e,x} + q_e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{B}] \right) \nabla_{\mathbf{p}} S_{e,x} \\ + \gamma_e \nabla B^x \nabla_{\mathbf{p}} (f_{e\uparrow} + f_{e\downarrow}) - \frac{2\gamma_e}{\hbar} \left(S_{e,y} B^z - (f_{e\uparrow} - f_{e\downarrow}) B^y \right) = 0, \end{aligned} \quad (27)$$

and

$$\begin{aligned} \partial_t S_y + \mathbf{v} \nabla_{\mathbf{r}} S_{e,y} + q_e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{B}] \right) \nabla_{\mathbf{p}} S_{e,y} \\ + \gamma_e \nabla B^y \nabla_{\mathbf{p}} (f_{e\uparrow} + f_{e\downarrow}) - \frac{2\gamma_e}{\hbar} \left((f_{e\uparrow} - f_{e\downarrow}) B^x - S_{e,x} B^z \right) = 0. \end{aligned} \quad (28)$$

Let us mention that S_x and S_y do not wear subindexes \uparrow and \downarrow . As we can see from definitions of S_x and S_y they are related to both projections spin-up Ψ_\uparrow and spin-down Ψ_\downarrow . Operators $\hat{\sigma}_i^x$ and $\hat{\sigma}_i^y$ mixing upper and lower components of N-rank spinor wave function. Whereas $\hat{\sigma}_i^z$ do not mix them giving $S_z(\mathbf{r}, \mathbf{p}, t) = f_{e\uparrow}(\mathbf{r}, \mathbf{p}, t) - f_{e\downarrow}(\mathbf{r}, \mathbf{p}, t)$. The full distribution of all electrons $f_e(\mathbf{r}, \mathbf{p}, t)$ is the sum of distribution functions of spin-up $f_{e\uparrow}(\mathbf{r}, \mathbf{p}, t)$ and spin-down $f_{e\downarrow}(\mathbf{r}, \mathbf{p}, t)$ electrons $f_e = f_{e\uparrow}(\mathbf{r}, \mathbf{p}, t) + f_{e\downarrow}(\mathbf{r}, \mathbf{p}, t)$.

Electromagnetic fields in the QHD equations presented above obey the Maxwell equations

$$\nabla \mathbf{E} = 4\pi e (n_i - n_{e\uparrow} - n_{e\downarrow}), \quad (29)$$

$$\nabla \mathbf{B} = 0, \quad (30)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}, \quad (31)$$

and

$$\begin{aligned} \nabla \times \mathbf{B} = \frac{1}{c} \partial_t \mathbf{E} + 4\pi \nabla \times \mathbf{M}_e \\ + \frac{4\pi}{c} (q_e \mathbf{j}_{e\uparrow} + q_e \mathbf{j}_{e\downarrow} + q_i \mathbf{j}_i), \end{aligned} \quad (32)$$

where $\mathbf{M}_e = \{\gamma_e \tilde{S}_{ex}, \gamma_e \tilde{S}_{ey}, \gamma_e (n_{e\uparrow} - n_{e\downarrow})\}$ is the magnetization of electrons in terms of hydrodynamic variables.

Material fields entering the Maxwell equations have the following relations to the distribution functions

$$n_a(\mathbf{r}, t) = \int f_a(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (33)$$

$$\mathbf{j}_a(\mathbf{r}, t) = \int \frac{\mathbf{p}}{m_a} f_a(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (34)$$

$$\tilde{S}_{ex}(\mathbf{r}, t) = \int S_{ex}(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (35)$$

and

$$\tilde{S}_{ey}(\mathbf{r}, t) = \int S_{ey}(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}. \quad (36)$$

Let us consider the spin density

$$\tilde{S}^\alpha(\mathbf{r}, t) = \int dR \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \psi^*(R, t) \hat{\sigma}_i^\alpha \psi(R, t), \quad (37)$$

proportional to the magnetization $\mathbf{M}_a(\mathbf{r}, t)$, usually used in the quantum hydrodynamics [4], [9], and [20]: $\mathbf{M}_a(\mathbf{r}, t) = \gamma_a \mathbf{S}_a(\mathbf{r}, t)$. Next integrating the spin distribution function over the momentum we get the spin density appearing in the quantum hydrodynamic equations [9], [20]

Here we describe explicit form of hydrodynamic spin density projections $\tilde{S}^\alpha(\mathbf{r}, t)$ for the simple single particle case to give a taste of the spin density structure, which reflects structure of the spin-distribution function. Here we need to represent both components of the spinor wave function as $\psi_s = a_s e^{i\phi_s}$. These quantities appear as follows $\tilde{S}_x = \psi^* \sigma_x \psi = \psi_\downarrow^* \psi_\uparrow + \psi_\uparrow^* \psi_\downarrow = 2a_\uparrow a_\downarrow \cos \Delta\phi$, $\tilde{S}_y = \psi^* \sigma_y \psi = i(\psi_\downarrow^* \psi_\uparrow - \psi_\uparrow^* \psi_\downarrow) = -2a_\uparrow a_\downarrow \sin \Delta\phi$, $\tilde{S}_z = \psi_\uparrow^* \psi_\uparrow - \psi_\downarrow^* \psi_\downarrow = a_\uparrow^2 - a_\downarrow^2$, where $\Delta\phi = \phi_\uparrow - \phi_\downarrow$. \tilde{S}_x , \tilde{S}_y , and \tilde{S}_z appear as mixed combinations of ψ_\uparrow and ψ_\downarrow . These quantities do not related to different species of electrons having different spin direction. \tilde{S}_x and \tilde{S}_y describe simultaneous evolution of both species.

IV. LINEARISED SET OF SEPARATED SPIN-UP AND SPIN-DOWN QUANTUM KINETIC EQUATIONS

Operator $[\mathbf{v}, \mathbf{e}_z] \partial_{\mathbf{p}}$ can be represented as $\frac{1}{m} \partial_\varphi$, where φ is the angle in the cylindrical coordinates in momentum space.

In equilibrium the set of kinetic equations (22), (23), (24), (27), and (28) has the following form

$$\partial_\varphi f_{0e\uparrow} = 0, \quad \partial_\varphi f_{0e\downarrow} = 0, \quad \partial_\varphi f_{0i} = 0, \quad (38)$$

$$\partial_\varphi S_{0e,x} = S_{0e,y}, \quad (39)$$

and

$$\partial_\varphi S_{0e,y} = -S_{0e,x}. \quad (40)$$

We have included that time and space derivatives of the distribution functions equal to zero. We have also included that electric field in equilibrium equals to zero as well. Equilibrium magnetic field equals to the external field directed parallel to the Z axis $B_x = B_y = 0$.

A. Equilibrium distributions

We consider degenerate electrons. In absence of the external magnetic field two electrons occupy each quantum state with momentum below the Fermi momentum

$$f_{0e} = \frac{2}{(2\pi\hbar)^3} \Theta(p_{Fe} - p), \quad (41)$$

where $p_{Fe} = (3\pi^2 n_{0e})^{1/3} \hbar$.

Distribution (41) is a spherically symmetric distribution, which does not contain dependence on angle variables θ , φ .

If we have degenerate electrons in an external magnetic field when occupation of spin-up and spin-down states are different. Under influence of an external magnetic field part of spin-up electrons change direction and transit to spin-down states. Thus instead of the Fermi step containing fully occupied (two electrons in a state) states, which is the unmodified Fermi step, we have two different Fermi steps for spin-up and spin-down electrons. The Fermi step for spin-up electrons is shorter than the unmodified Fermi step, whereas The Fermi step for spin-down electrons is longer than the unmodified Fermi step. Equilibrium distribution function for each subspecies of electrons are

$$f_{0s} = \frac{1}{(2\pi\hbar)^3} \Theta(p_{Fs} - p), \quad (42)$$

where $p_{Fs} = (6\pi^2 n_{0s})^{1/3} \hbar$, and $s = \uparrow$, or \downarrow .

Distribution function of all electrons is the sum of $f_{0\uparrow}$ and $f_{0\downarrow}$, hence

$$f_{0e} = \frac{1}{(2\pi\hbar)^3} [\theta(p_{F\uparrow} - p) + \theta(p_{F\downarrow} - p)], \quad (43)$$

which pass into (41) at $B_0 \rightarrow 0$.

We consider two limits for ions: classic low temperature ions and degenerate ions.

For classic ions we consider the Maxwell distribution function for equilibrium distribution

$$f_{0i}(\mathbf{p}) = \frac{n_{0i}}{(\sqrt{2\pi m_i T_i})^3} \exp\left(-\frac{\mathbf{p}^2}{2m_i T_i}\right), \quad (44)$$

where T_i is the temperature of classic ions in units of energy, and $\mathbf{p} = m\mathbf{v}$.

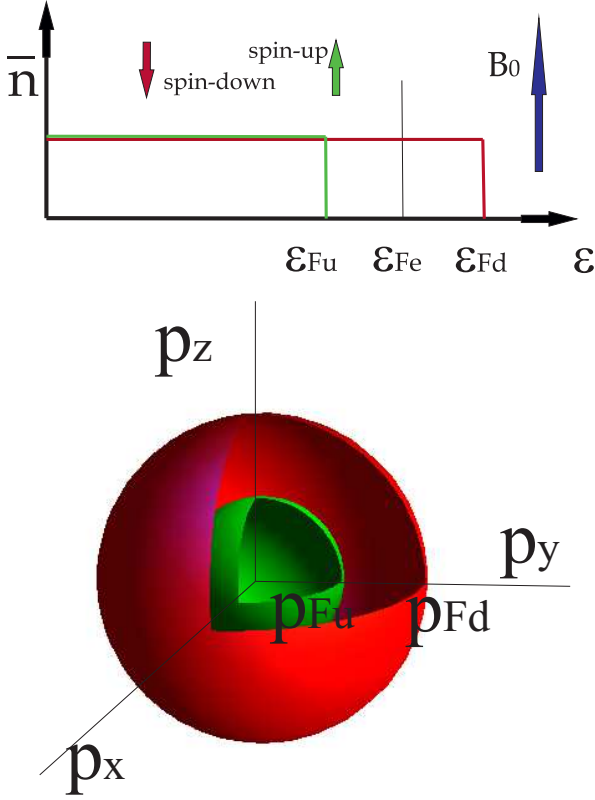


FIG. 1: (Color online) The figure shows distribution functions \bar{n} of degenerate spin-up and spin-down electrons being in external magnetic field. This distribution function gives average occupation number of quantum states with different energies.

For degenerate ions we have

$$f_{0i} = \frac{2}{(2\pi\hbar)^3} \Theta(p_{Fi} - p). \quad (45)$$

We neglect change of ion occupation number for magnetised ions.

From equations (39) and (40) we find general form of dependence of equilibrium spin distribution functions on momentum

$$S_{0x} = C(p_{\parallel}, p_{\perp}) \cos(\varphi + \varphi_0), \quad S_{0y} = C(p_{\parallel}, p_{\perp}) \sin(\varphi + \varphi_0). \quad (46)$$

Further analysis leads to the following explicit form of equilibrium spin distribution functions

$$S_{0x} = \frac{1}{(2\pi\hbar)^3} \left(\Theta(p_{Fu} - p) + \Theta(p_{Fd} - p) \right) \cos(\varphi + \varphi_0), \quad (47)$$

$$S_{0y} = \frac{1}{(2\pi\hbar)^3} \left(\Theta(p_{Fu} - p) + \Theta(p_{Fd} - p) \right) \sin(\varphi + \varphi_0). \quad (48)$$

Let us mention that corresponding equilibrium hydrodynamic spin densities equal to zero

$$\int S_{0x}(\mathbf{p}) d\mathbf{p} = 0, \quad (49)$$

and

$$\int S_{0y}(\mathbf{p}) d\mathbf{p} = 0, \quad (50)$$

as it should be. These integrals equal to zero due to integration over the angle φ .

B. Linearised kinetic equations

Now we are ready to present linearised kinetic equations.

Linearised kinetic equation for spin-up electrons reads as

$$\begin{aligned} & \partial_t \delta f_{e\uparrow} + \mathbf{v} \cdot \nabla_{\mathbf{r}} \delta f_{e\uparrow} \\ & + q_e \left(\delta \mathbf{E} + \frac{1}{c} [\mathbf{v}, \delta \mathbf{B}] \right) \cdot \nabla_{\mathbf{p}} f_{0e\uparrow} + q_e \frac{1}{c} [\mathbf{v}, \mathbf{B}_0] \cdot \nabla_{\mathbf{p}} \delta f_{e\uparrow} \\ & + \gamma_e \nabla \delta B^z \cdot \nabla_{\mathbf{p}} f_{0e\uparrow} + \frac{\gamma_e}{2} \left(\partial_{\alpha} \delta B^x \cdot \nabla_{\mathbf{p}\alpha} S_{0e,x} \right. \\ & \left. + \nabla \delta B^y \cdot \nabla_{\mathbf{p}} S_{0e,y} \right) = \frac{\gamma_a}{\hbar} \left(S_{0e,x} \delta B_y - S_{0e,y} \delta B_x \right). \end{aligned} \quad (51)$$

Linearised kinetic equation for spin-down electrons appears as

$$\begin{aligned} & \partial_t \delta f_{e\downarrow} + \mathbf{v} \cdot \nabla_{\mathbf{r}} \delta f_{e\downarrow} \\ & + q_e \left(\delta \mathbf{E} + \frac{1}{c} [\mathbf{v}, \delta \mathbf{B}] \right) \cdot \nabla_{\mathbf{p}} f_{0e\downarrow} + q_e \frac{1}{c} [\mathbf{v}, \mathbf{B}_0] \cdot \nabla_{\mathbf{p}} \delta f_{e\downarrow} \\ & - \gamma_e \nabla_{\alpha} \delta B^z \cdot \nabla_{\mathbf{p}\alpha} f_{0e\downarrow} + \frac{\gamma_e}{2} \left(\nabla \delta B^x \cdot \nabla_{\mathbf{p}} S_{0e,x} \right. \\ & \left. + \nabla \delta B^y \cdot \nabla_{\mathbf{p}} S_{0e,y} \right) = -\frac{\gamma_a}{\hbar} \left(S_{0e,x} \delta B_y - S_{0e,y} \delta B_x \right). \end{aligned} \quad (52)$$

Linearised kinetic equation for ions has the following form

$$\begin{aligned} & \partial_t \delta f_i + \mathbf{v} \cdot \nabla_{\mathbf{r}} \delta f_i \\ & + q_i \left(\delta \mathbf{E} + \frac{1}{c} [\mathbf{v}, \delta \mathbf{B}] \right) \cdot \nabla_{\mathbf{p}} f_{0i} + q_i \frac{1}{c} [\mathbf{v}, \mathbf{B}_0] \cdot \nabla_{\mathbf{p}} \delta f_i = 0. \end{aligned} \quad (53)$$

Linearised kinetic equation for x-projection of the spin-distribution function of electrons

$$\begin{aligned}
& \partial_t \delta S_{e,x} + \mathbf{v} \cdot \nabla_{\mathbf{r}} \delta S_{e,x} \\
& + q_e \left(\delta \mathbf{E} + \frac{1}{c} [\mathbf{v}, \delta \mathbf{B}] \right) \cdot \nabla_{\mathbf{p}} S_{0e,x} + q_e \frac{1}{c} [\mathbf{v}, \mathbf{B}_0] \cdot \nabla_{\mathbf{p}} \delta S_{e,x} \\
& + \gamma_e \nabla \delta B^x \cdot \nabla_{\mathbf{p}} (f_{0e\uparrow} + f_{0e\downarrow}) \\
& = \frac{2\gamma_e}{\hbar} \left(\delta S_{e,y} B_{0z} + S_{0e,y} \delta B_z - (f_{0e\uparrow} - f_{0e\downarrow}) \delta B^y \right), \quad (54)
\end{aligned}$$

and linearised kinetic equation for y-projection of the spin-distribution function of electrons

$$\begin{aligned}
& \partial_t \delta S_y + \mathbf{v} \cdot \nabla_{\mathbf{r}} \delta S_{e,y} \\
& + q_e \left(\delta \mathbf{E} + \frac{1}{c} [\mathbf{v}, \delta \mathbf{B}] \right) \cdot \nabla_{\mathbf{p}} S_{0e,y} + \frac{1}{c} [\mathbf{v}, \mathbf{B}_0] \cdot \nabla_{\mathbf{p}} \delta S_{e,y} \\
& + \gamma_e \nabla \delta B^y \cdot \nabla_{\mathbf{p}} (f_{0e\uparrow} + f_{0e\downarrow}) \\
& = \frac{2\gamma_e}{\hbar} \left((f_{0e\uparrow} - f_{0e\downarrow}) \delta B^x - S_{0e,x} \delta B_z - \delta S_{e,x} B_{0z} \right). \quad (55)
\end{aligned}$$

Equations of matter evolution (51)-(55) are coupled with equations of electromagnetic field

$$\nabla \delta \mathbf{E} = 4\pi \sum_{a=u,d,i} q_a \int \delta f_a(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (56)$$

$$\nabla \delta \mathbf{B} = 0, \quad (57)$$

$$\nabla \times \delta \mathbf{E} = -\frac{1}{c} \partial_t \delta \mathbf{B}, \quad (58)$$

and

$$\begin{aligned}
& \nabla \times \delta \mathbf{B} = \frac{1}{c} \partial_t \delta \mathbf{E} + 4\pi \nabla \times \delta \mathbf{M}_e \\
& + \frac{4\pi}{c} \sum_{a=u,d,i} q_a \int \frac{\mathbf{p}}{m_a} \delta f_a(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (59)
\end{aligned}$$

where

$$\delta M_x = \gamma_e \int \delta S_{e,x}(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (60)$$

$$\delta M_y = \gamma_e \int \delta S_{e,y}(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (61)$$

and

$$\delta M_z = \gamma_e \int [\delta f_{\uparrow}(\mathbf{r}, \mathbf{p}, t) - \delta f_{\downarrow}(\mathbf{r}, \mathbf{p}, t)] d\mathbf{p}. \quad (62)$$

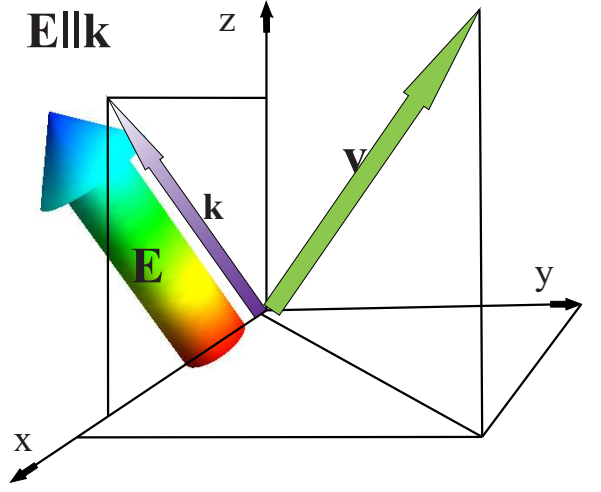


FIG. 2: (Color online) The figure shows velocity, wave vector, and electric field in oblique propagating longitudinal waves.

C. Small amplitude perturbations propagating parallel to the external magnetic field

Equilibrium condition is described by the non-zero concentrations $n_{0\uparrow}$, $n_{0\downarrow}$, $n_0 = n_{0\uparrow} + n_{0\downarrow}$, S_{0x} , S_{0y} , and an external magnetic field $\mathbf{B}_{ext} = B_0 \mathbf{e}_z$, but $\mathbf{E}_0 = 0$. Assuming that perturbations are monochromatic

$$\begin{pmatrix} \delta f_{e\uparrow} \\ \delta f_{e\downarrow} \\ \delta f_i \\ \delta \mathbf{E} \\ \delta \mathbf{B} \\ \delta S_x \\ \delta S_y \end{pmatrix} = \begin{pmatrix} F_{A\uparrow} \\ F_{A\downarrow} \\ F_{Ai} \\ \mathbf{E}_A \\ \mathbf{B}_A \\ S_{Ax} \\ S_{Ay} \end{pmatrix} e^{-i\omega t + i\mathbf{k}\mathbf{r}}, \quad (63)$$

we get a set of linear algebraic equations relatively to $F_{A\uparrow}$, $F_{A\downarrow}$, F_{Ai} , $V_{A\uparrow}$, $V_{A\downarrow}$, \mathbf{E}_A , \mathbf{B}_A , S_{Ax} , and S_{Ay} . Condition of existence of nonzero solutions for amplitudes of perturbations gives us a dispersion equation.

Difference of spin-up and spin-down concentrations of electrons $\Delta n = n_{0\uparrow} - n_{0\downarrow}$ is caused by external magnetic field. Since electrons are negative their spins get preferable direction opposite to the external magnetic field $\frac{\Delta n}{n_0} = \tanh(\frac{\gamma_e B_0}{T_e}) = -\tanh(\frac{|\gamma_e| B_0}{T_e})$. Here we consider temperature in units of energy, so we do not use the Boltzmann constant.

We consider plasmas in the uniform constant external magnetic field. We see that in linear approach numbers of electrons of each species conserves.

Linearised set of kinetic equations has rather complex form, but we follow results of Refs. [1] and [3], hence we are focused on properties of the longitudinal waves. This assumption makes analysis more simple. We should mention that waves in magnetised plasmas are not longitudinal in most cases. An exceptional regime supporting propagation of longitudinal waves is limit of wave propagation parallel to external magnetic field. Thus this is

the main area of our research. However we obtain an approximate dispersion equation for longitudinal waves in regime of oblique wave propagation. Longitudinal waves require $\mathbf{E} \parallel \mathbf{k}$. As a consequence we obtain $\delta\mathbf{B} = 0$.

V. DISPERSION EQUATION FOR LONGITUDINAL WAVES: GENERAL FORM

General form of dispersion equation for oblique propagating longitudinal waves in separated spin evolution model appears as

$$\begin{aligned}
& 1 + \frac{4\pi^2 e^2}{k} \left\{ \sum_{s=u,d} \sum_{n=-\infty}^{+\infty} \frac{p_{Fs}^2}{(2\pi\hbar)^3} \times \right. \\
& \times \int_0^\pi \sin\theta d\theta \frac{J_n\left(\frac{k_x v_{Fs}}{\Omega_s} \sin\theta\right)}{k_z v_{Fs} \cos\theta - \omega + n\Omega_s} \times \\
& \times \left[2 \cos\alpha \cos\theta J_n\left(\frac{k_x v_{Fs}}{\Omega_s} \sin\theta\right) \right. \\
& \left. \left. + \sin\alpha \sin\theta \left(J_{n+1}\left(\frac{k_x v_{Fs}}{\Omega_s} \sin\theta\right) + J_{n-1}\left(\frac{k_x v_{Fs}}{\Omega_s} \sin\theta\right) \right) \right] \right\} \\
& + \sum_{n=-\infty}^{+\infty} \frac{n_{0i}}{(2\pi m_i T_i)^{\frac{3}{2}}} \frac{1}{2\pi T_i} \int d\mathbf{p} \frac{J_n\left(\frac{k_x v_{Ti}}{\Omega_i} \sin\theta\right) e^{-\frac{\mathbf{p}^2}{2m_i T_i}}}{k_z v_z - \omega + n\Omega_i} \times \\
& \times \left(2 J_n\left(\frac{k_x v_{Ti}}{\Omega_i} \sin\theta\right) v_z \cos\alpha \right. \\
& \left. + \left[J_{n+1}\left(\frac{k_x v_{Ti}}{\Omega_i} \sin\theta\right) + J_{n-1}\left(\frac{k_x v_{Ti}}{\Omega_i} \sin\theta\right) \right] \right) \Bigg\} = 0, \tag{64}
\end{aligned}$$

where $v_z = v \cos\theta$, $v_\perp = v \sin\theta$, $k_z = k \cos\alpha$, $k_x = k \sin\alpha$, and $J_n(x)$ are the Bessel functions.

Dispersion equation for longitudinal waves propagating parallel to external field in magnetised plasmas rather simpler, in this limit we have $\mathbf{k} \parallel \mathbf{B}_0$, consequently we obtain $\alpha = 0$, $k_x = 0$, $k_z = k$. This assumption leads to more simplification $J_n(0) = 0$ if $n \neq 0$, and $J_0(0) = 1$.

After all these simplifications we find

$$\begin{aligned}
& 1 + \frac{8\pi^2 e^2}{k^2} \left(\sum_{s=u,d} \frac{m_s^2 v_{Fs}}{(2\pi\hbar)^3} \left(2 + \omega \int_0^\pi \frac{\sin\theta d\theta}{k v_{Fs} \cos\theta - \omega} \right) \right. \\
& \left. + \frac{1}{2\pi T_i} \frac{n_{0i}}{(2\pi m_i T_i)^{\frac{1}{2}}} \left((2\pi m_i T_i)^{\frac{1}{2}} + \omega \int \frac{e^{-\frac{p_z^2}{2m_i T_i}}}{k v_z - \omega} dp_z \right) \right) = 0. \tag{65}
\end{aligned}$$

For the Maxwell distribution in equilibrium state we meet the following integral in the dispersion equation

$$\begin{aligned}
Z(\alpha) &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{\exp(-\xi^2)}{\xi - \alpha} d\xi \\
&= \frac{1}{\sqrt{\pi}} \left[P \int_{-\infty}^{+\infty} \frac{\exp(-\xi^2)}{\xi - \alpha} d\xi \right] + i\sqrt{\pi} \exp(-\alpha^2), \tag{66}
\end{aligned}$$

where $\alpha = \frac{\omega}{k v_T}$ with $v_T \equiv \sqrt{\frac{T}{m}}$, and the symbol P denotes the principle part of the integral.

Let us present assumptions of formula (66). At $\alpha \gg 1$ we have

$$Z(\alpha) \simeq -\frac{1}{\alpha} \left(1 + \frac{1}{2\alpha^2} + \frac{3}{4\alpha^4} + \dots \right) + i\sqrt{\pi} \exp(-\alpha^2). \tag{67}$$

This approximate formula will be applied below at description of classic ion contribution in spectrum of plasmas.

VI. SPECTRUM OF LONGITUDINAL WAVES PROPAGATING PARALLEL TO EXTERNAL FIELD IN MAGNETISED SEPARATED SPIN-UP AND SPIN-DOWN QUANTUM PLASMAS

Taking integrals over angle θ and p_z in equation (65) we get the following explicit form of dispersion equations in regimes of classic and degenerate ions.

Classic ions

Degenerate electrons give, in dispersion equation, well-known logarithmic term. Since we have two species of electrons we obtain two logarithmic terms:

$$\begin{aligned}
1 &= \frac{3}{2} \frac{\omega_{Lu}^2}{v_{Fu}^2 k^2} \left(\frac{\omega}{k v_{Fu}} \ln \frac{\omega + k v_{Fu}}{\omega - k v_{Fu}} - 2 \right) \\
&+ \frac{3}{2} \frac{\omega_{Ld}^2}{v_{Fd}^2 k^2} \left(\frac{\omega}{k v_{Fd}} \ln \frac{\omega + k v_{Fd}}{\omega - k v_{Fd}} - 2 \right) + \frac{\omega_{Li}^2}{\omega^2} \left(1 + 3 \frac{k^2 v_{Ti}^2}{\omega^2} \right) \\
&- \sqrt{\frac{\pi}{2}} i \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \frac{\omega}{k v_{Ti}} \exp\left(-\frac{\omega^2}{2k^2 v_{Ti}^2}\right). \tag{68}
\end{aligned}$$

Quantum degenerate ions

If ions are degenerate as well as electrons we have three similar logarithmic terms

$$1 = \sum_{a=u,d,i} \frac{3}{2} \frac{\omega_{La}^2}{v_{Fa}^2 k^2} \left(\frac{\omega}{k v_{Fa}} \ln \frac{\omega + k v_{Fa}}{\omega - k v_{Fa}} - 2 \right). \tag{69}$$

Below we present approximate formulas we apply to solve dispersion equations analytically.

At $\omega \gg k v_a$ one finds well-known expansion

$$\frac{\omega}{k v_a} \ln \frac{\omega + k v_a}{\omega - k v_a} = 2 \left(1 + \frac{1}{3} \frac{k^2 v_a^2}{\omega^2} + \frac{1}{5} \frac{k^4 v_a^4}{\omega^4} \right). \tag{70}$$

At $\omega \ll k v_a$ we obtain another well-known expansion

$$\frac{\omega}{k v_a} \ln \frac{\omega + k v_a}{\omega - k v_a} = -\pi i \frac{\omega}{k v_a} + 2 \frac{\omega^2}{k^2 v_a^2} \left(1 + \frac{1}{3} \frac{\omega^2}{k^2 v_a^2} \right). \tag{71}$$

A. Langmuir wave

Dispersion equation for high frequency regime at classic ions

$$1 = \frac{\omega_{Ld}^2}{\omega^2} \left(1 + \frac{3}{5} \frac{k^2 v_{Fd}^2}{\omega^2} \right) + \frac{\omega_{Lu}^2}{\omega^2} \left(1 + \frac{3}{5} \frac{k^2 v_{Fu}^2}{\omega^2} \right) + \frac{\omega_{Li}^2}{\omega^2} \left(1 + 3 \frac{k^2 v_{Ti}^2}{\omega^2} \right) - \sqrt{\frac{\pi}{2}} \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \frac{\omega}{k v_{Ti}} \exp\left(-\frac{\omega^2}{2k^2 v_{Ti}^2}\right). \quad (72)$$

Dispersion equation for high frequency regime at classic ions

$$1 = \frac{\omega_{Lu}^2}{\omega^2} \left(1 + \frac{3}{5} \frac{k^2 v_{Fu}^2}{\omega^2} \right) + \frac{\omega_{Li}^2}{\omega^2} \left(1 + 3 \frac{k^2 v_{Ti}^2}{\omega^2} \right). \quad (73)$$

In general case the frequency of excitations appears in complex form $\omega = \omega_R + i\omega_{Im}$.

Spectrums of the Langmuir waves are

$$\omega_R^2 = (\omega_{Ld}^2 + \omega_{Lu}^2 + \omega_{Li}^2) + \frac{3}{5} k^2 \left(\frac{n_{0u}}{n_{0e}} v_{Fu}^2 + \frac{n_{0d}}{n_{0e}} v_{Fd}^2 + \frac{m_e}{m_i} v_{Fi}^2 \right) \quad (74)$$

for degenerate ions, and

$$\omega_R^2 = (\omega_{Ld}^2 + \omega_{Lu}^2 + \omega_{Li}^2) + \frac{3}{5} k^2 \left(\frac{n_{0u}}{n_{0e}} v_{Fu}^2 + \frac{n_{0d}}{n_{0e}} v_{Fd}^2 + \frac{5}{3} \frac{m_e}{m_i} v_{Ti}^2 \right), \quad (75)$$

with

$$\omega_{Im} = \frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \frac{\omega_{Le}^2}{k v_{Ti}} \exp\left(-\frac{\omega_{Le}^2}{2k^2 v_{Ti}^2}\right) \quad (76)$$

for classic electrons.

In formulae (74) and (75) sum of partial Langmuir frequencies $\omega_{Ld}^2 + \omega_{Lu}^2$ gives full Langmuir frequency of electrons $\omega_{Le}^2 = \omega_{Ld}^2 + \omega_{Lu}^2$, since $n_{0e} = n_{0u} + n_{0d}$.

The Langmuir wave in degenerate electron gas does not have collisionless damping $\omega_{Im} = 0$ if ions are degenerate as well. In regime of classic (Maxwellian) ions, there is small damping of the Langmuir waves in degenerate electron gas.

In Ref. [1] we have obtained $\omega^2 = (\omega_{Ld}^2 + \omega_{Lu}^2) + \frac{1}{3} k^2 (\frac{n_{0u}}{n_{0e}} v_{Fu}^2 + \frac{n_{0d}}{n_{0e}} v_{Fd}^2)$. We can see that pressure term has different coefficient. This difference appeared due

to application of the Fermi pressure (for spin-up and spin-down separately) as an equation of state $P_{Fs} = \frac{1}{5} \frac{(6\pi^2)^{\frac{2}{3}} n_s^{\frac{5}{3}} \hbar^2}{m}$. The Fermi pressure gives the equation of state for equilibrium, whereas we considered perturbations of an equilibrium state. To cancel the difference between hydrodynamic and kinetics of small perturbations in three dimensional plasmas we can write down the following modified equation of state $P_{ms} = \frac{1}{5} \frac{m v_{Fs}^2}{n_{0s}^2} n_s^3$ (see Ref. [13] formula 99), where n_s is the full concentration of particles with s spin projection on z direction, and n_{0s} is the equilibrium concentration of particles with s spin projection. This formula gives the Fermi pressure in equilibrium $n = n_0$, and it gives spectrum coinciding with results of kinetic theory of degenerate electron gas.

Let us represent the real part of the Langmuir spectrum in approximate, and more explicit form. We present this spectrum in terms of conventional variables ω_{Le} , v_{Fe} , and Δn , hence we have

$$\omega^2 = \omega_{Le}^2 + \frac{3}{10} k^2 v_{Fe}^2 \left[\left(1 - \frac{\Delta n}{n_{0e}} \right)^{\frac{5}{3}} + \left(1 + \frac{\Delta n}{n_{0e}} \right)^{\frac{5}{3}} \right]. \quad (77)$$

This dependence on Δn corresponds to results obtained in Refs. [5], [21].

B. Spin-electron acoustic waves

In this subsection we present one of main results of this paper. We present the kinetic analysis of the spin-electron acoustic waves. At hydrodynamic description we were able to get spectrum of the SEAWs at all wave vectors [1], [2]. Here we can get an analytical solution for intermediate frequencies described below (see conditions (78) and (86)). Nevertheless, the quantum kinetics allows us to study the Landau damping of the SEAWs.

Regime of high spin polarisation allows to perform analytic consideration of the spin-electron acoustic wave spectrum.

1. SEAW: Classic ions

Part of spectrum of spin-electron acoustic wave can be derived at the following conditions

$$k v_{Ti}, k v_{Fu} \ll \omega \ll k v_{Fd}. \quad (78)$$

Dispersion equation (68) simplifies at conditions (78). Its simple form appears as

$$1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \left(1 + \frac{\pi}{2} \frac{\omega}{k v_{Fd}} - \frac{\omega^2}{k^2 v_{Fd}^2} \right) = \frac{\omega_{Lu}^2}{\omega^2} \left(1 + \frac{3}{5} \frac{k^2 v_{Fu}^2}{\omega^2} \right) + \frac{\omega_{Li}^2}{\omega^2} \left(1 + 3 \frac{k^2 v_{Ti}^2}{\omega^2} \right)$$

$$-\sqrt{\frac{\pi}{2}} \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \frac{\omega}{k v_{Ti}} \exp\left(-\frac{\omega^2}{2k^2 v_{Ti}^2}\right). \quad (79)$$

In major order equation (79) gives the following spectrum

$$\omega_{R0}^2 = \frac{(\omega_{Lu}^2 + \omega_{Li}^2)}{1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2}}. \quad (80)$$

Including terms of second order we obtain more general dispersion dependence

$$\omega_R^2 = \frac{\omega_{Lu}^2 \left(1 + \frac{3}{5} \frac{k^2 v_{Fu}^2}{\omega_{R0}^2}\right) + \omega_{Li}^2 \left(1 + \frac{3}{5} \frac{k^2 v_{Fi}^2}{\omega_{R0}^2}\right)}{1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} - 3 \frac{\omega_{Ld}^2 \omega_{R0}^2}{k^4 v_{Fd}^4}}. \quad (81)$$

We also obtain imaginary part of the frequency giving Landau damping of the SEAW

$$\omega_{Im} = \frac{1}{2} \omega_R \frac{\frac{3\pi}{2} \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \frac{\omega_{R0}}{k v_{Fd}} + \sqrt{\frac{\pi}{2}} \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \frac{\omega_{R0}}{k v_{Ti}} \exp\left(-\frac{\omega_{R0}^2}{2k^2 v_{Ti}^2}\right)}{1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} - 3 \frac{\omega_{Ld}^2 \omega_{R0}^2}{k^4 v_{Fd}^4}}. \quad (82)$$

In long-wavelength regime $\omega_{Ld} \gg k v_{Fd}$ formula (80) simplifies to

$$\omega_{R0}^2 = \frac{1}{3} k^2 v_{Fd}^2 \frac{(\omega_{Lu}^2 + \omega_{Li}^2)}{\omega_{Ld}^2}. \quad (83)$$

At intermediate spin polarisation $\omega_{Lu}^2 \gg \omega_{Li}^2$ we can neglect ion contribution in formula (83) and find

$$\omega_{R0}^2 = \frac{1}{3} \frac{n_{0u}}{n_{0d}} k^2 v_{Fd}^2. \quad (84)$$

2. SEAW: Degenerate ions

Dispersion equation for the SEAW has form of

$$1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \left(1 + \frac{\pi}{2} \frac{\omega}{k v_{Fd}} - \frac{\omega^2}{k^2 v_{Fd}^2}\right) = \frac{\omega_{Lu}^2}{\omega^2} \left(1 + \frac{3}{5} \frac{k^2 v_{Fu}^2}{\omega^2}\right) + \frac{\omega_{Li}^2}{\omega^2} \left(1 + \frac{3}{5} \frac{k^2 v_{Fi}^2}{\omega^2}\right). \quad (85)$$

Equation (85) arises at conditions

$$k v_{Fi}, k v_{Fu} \ll \omega \ll k v_{Fd}. \quad (86)$$

Equation (85) gives spectrum of the SEAWs

$$\omega_{R0}^2 = \frac{(\omega_{Lu}^2 + \omega_{Li}^2)}{1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2}}. \quad (87)$$

Landau damping of the SEAW is found to be

$$\omega_{Im} = \frac{1}{2} \omega_R \frac{\frac{3\pi}{2} \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \frac{\omega_{R0}}{k v_{Fd}}}{1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} - 3 \frac{\omega_{Ld}^2 \omega_{R0}^2}{k^4 v_{Fd}^4}}. \quad (88)$$

At $\omega_{Ld}^2 \gg k^2 v_{Fd}^2$ we find simplification of formula (88) $\omega_{Im} = \frac{\pi}{4} \frac{\omega_{R0}}{k v_{Fd}} \omega_{R0} \ll \omega_{R0}$.

In opposite limit $\omega_{Ld}^2 \ll k^2 v_{Fd}^2$ the denominator in formula (88) equals to 1 and we have $\omega_{Im} = \frac{3\pi}{4} \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \frac{\omega_{R0}}{k v_{Fd}} \omega_{R0} \ll \omega_{R0}$.

So, the Landau damping of the SEAWs always smaller than frequency of the wave. Thus we have found that the SEAW is a weakly damped wave.

Including smaller corrections in equation (85) we can find generalisation of formula (87)

$$\omega_R^2 = \frac{\omega_{Lu}^2 \left(1 + \frac{3}{5} \frac{k^2 v_{Fu}^2}{\omega_{R0}^2}\right) + \omega_{Li}^2 \left(1 + \frac{3}{5} \frac{k^2 v_{Fi}^2}{\omega_{R0}^2}\right)}{1 + 3 \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} - 3 \frac{\omega_{Ld}^2 \omega_{R0}^2}{k^4 v_{Fd}^4}}. \quad (89)$$

Formula (80) coincides with the result for Maxwellian ions (80). Hence its long-wavelength limit ($\omega_{Ld} \gg k v_{Fd}$) coincides with formula (83).

3. SEAW: Discussion

Formulae (80) and (87) can be rewritten in terms of ω_{Le} and v_{Fe} . This representation explicitly shows contribution of mismatch of the Fermi surfaces of spin-up and spin-down electrons.

$$\omega_{R0}^2 = \frac{\frac{1}{2} (1 - \frac{\Delta n}{n_{0e}}) \omega_{Le}^2 + \omega_{Li}^2}{1 + \frac{3}{2} \frac{\omega_{Le}^2}{k^2 v_{Fe}^2} (1 + \frac{\Delta n}{n_{0e}})^{\frac{1}{3}}}. \quad (90)$$

At $\omega_{Ld} \gg k v_{Fd}$ and $\omega_{Lu}^2 \gg \omega_{Li}^2$ formula (90) simplifies and we find

$$\omega_{R0}^2 = \frac{1}{3} \frac{(1 - \frac{\Delta n}{n_{0e}})}{(1 + \frac{\Delta n}{n_{0e}})^{\frac{1}{3}}} \cdot k^2 v_{Fe}^2. \quad (91)$$

In this subsection we work under condition $v_{Fd} \gg v_{Fu} \Rightarrow 2 > 1 + \frac{\Delta n}{n_{0e}} \gg 1 + \frac{\Delta n}{n_{0e}} \Rightarrow \Delta n$ comparable with n_0 . Thus we conclude that phase velocity of the SEAW given by formula (91) considerably less than the electron Fermi velocity $\omega_{R0} \approx \frac{1}{\sqrt{6}} \sqrt{1 - \frac{\Delta n}{n_{0e}}} k v_{Fe} \ll k v_{Fe}$.

C. SEAW: Regime of phase velocity near the Fermi velocity of spin-up electrons v_{Fu}

Let us consider the limit $\omega \rightarrow k v_{Fu}$, which is the low frequency analog of the zeroth sound. In this case we

can present frequency of oscillations as $\omega = kv_{Fu} + \delta\omega$. Dispersion equation arises as

$$1 = \frac{3}{2} \frac{\omega_{Lu}^2}{k^2 v_{Fu}^2} \left[\frac{\omega}{kv_{Fu}} \ln \left(\frac{2kv_{Fu}}{\delta\omega} \right) - 2 \right] - \frac{3}{2} \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \left(2 + \frac{n_{0u}^{\frac{1}{3}}}{n_{0d}^{\frac{1}{3}}} \ln \frac{v_{Fd} - v_{Fu}}{v_{Fd} + v_{Fu}} + \pi \nu \left(\frac{n_{0u}}{n_{0d}} \right)^{\frac{1}{3}} \right). \quad (92)$$

Corresponding dispersion dependence arises as

$$\omega = kv_{Fu} \left\{ 1 - 2 \frac{v_{Fd} + v_{Fu}}{v_{Fd} - v_{Fu}} \times \exp \left[-2 - \frac{2}{3} \frac{k^2 v_{Fu}^2}{\omega_{Lu}^2} - 2 \left(\frac{n_{0d}}{n_{0u}} \right)^{\frac{1}{3}} \right] \right\}. \quad (93)$$

D. Ion-acoustic wave

1. Classic ions

Ion-acoustic waves exist at the following conditions

$$kv_{Ti} \ll \omega \ll kv_{Fu}, kv_{Fd}. \quad (94)$$

Dispersion equation for ion acoustic waves with classic ions has the following form

$$1 + 3 \left(\frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} + \frac{\omega_{Lu}^2}{k^2 v_{Fu}^2} \right) + \frac{3}{2} \pi \nu \left(\frac{\omega_{Ld}^2}{k^3 v_{Fd}^3} + \frac{\omega_{Lu}^2}{k^3 v_{Fu}^3} \right) = \frac{\omega_{Li}^2}{\omega^2} - \sqrt{\frac{\pi}{2}} \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \frac{\omega}{kv_{Ti}} \exp \left(-\frac{\omega^2}{2k^2 v_{Ti}^2} \right). \quad (95)$$

Equation (95) gives dispersion dependence of ion-acoustic waves

$$\omega_R^2 = \frac{\omega_{Li}^2}{1 + 3 \left(\frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} + \frac{\omega_{Lu}^2}{k^2 v_{Fu}^2} \right)}. \quad (96)$$

If $\omega_{Ls} \gg kv_{Fs}$ we find long-wavelength limit of dispersion dependence (96)

$$\omega_R^2 = \frac{1}{3} k^2 \frac{v_{Fd}^2 v_{Fu}^2 \omega_{Li}^2}{v_{Fd}^2 \omega_{Lu}^2 + v_{Fu}^2 \omega_{Ld}^2}. \quad (97)$$

Imaginary part of frequency of ion-acoustic waves at Maxwellian ion appears as

$$\omega_{Im} = -\frac{3\pi}{4} \frac{\omega_R^4}{\omega_{Li}^2} \left(\frac{\omega_{Ld}^2}{k^3 v_{Fd}^3} + \frac{\omega_{Lu}^2}{k^3 v_{Fu}^3} \right) - \frac{1}{2} \sqrt{\frac{\pi}{2}} \left(\frac{\omega_R}{kv_{Ti}} \right)^3 \omega_R \exp \left(-\frac{\omega_R^2}{2k^2 v_{Ti}^2} \right). \quad (98)$$

2. Degenerate ions

Conditions of existence of the ion-acoustic waves in plasmas of degenerate electrons and ions are

$$kv_{Fi} \ll \omega \ll kv_{Fu}, kv_{Fd}. \quad (99)$$

In this regime we can obtain the dispersion equation

$$1 + 3 \left(\frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} + \frac{\omega_{Lu}^2}{k^2 v_{Fu}^2} \right) + \frac{3}{2} \pi \nu \left(\frac{\omega_{Ld}^2}{k^3 v_{Fd}^3} + \frac{\omega_{Lu}^2}{k^3 v_{Fu}^3} \right) = \frac{\omega_{Li}^2}{\omega^2}. \quad (100)$$

Equation (100) gives the following solution in leading order on small parameters

$$\omega_R^2 = \frac{\omega_{Li}^2}{1 + 3 \left(\frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} + \frac{\omega_{Lu}^2}{k^2 v_{Fu}^2} \right)}. \quad (101)$$

At $\omega_{Ls} \gg kv_{Fs}$ (long-wavelength regime) we find simplification of solution (101) as follows

$$\omega_R^2 = \frac{1}{3} k^2 \frac{v_{Fd}^2 v_{Fu}^2 \omega_{Li}^2}{v_{Fd}^2 \omega_{Lu}^2 + v_{Fu}^2 \omega_{Ld}^2}. \quad (102)$$

Including smaller corrections to the spectrum (101) find decrement of the Landau damping for ion-acoustic waves

$$\omega_{Im} = -\frac{3\pi}{4} \frac{\omega_R^4}{\omega_{Li}^2} \left(\frac{\omega_{Ld}^2}{k^3 v_{Fd}^3} + \frac{\omega_{Lu}^2}{k^3 v_{Fu}^3} \right). \quad (103)$$

3. Ion acoustic waves: Discussion

At $kv_{Fs} \gg \omega_{Ls}$ formula (101) gives well-known limit $\omega_R^2 = \omega_{Li}^2$.

Next let us consider formula (102), which has been obtained from formula (101) in regime of long-wavelength $\omega_{Ls} \gg kv_{Fs}$, in more explicit form

$$\omega_R^2 = \frac{2}{3} \frac{m_e}{m_i} k^2 v_{Fe}^2 \frac{1}{\left(1 - \frac{\Delta n}{n_{0e}}\right)^{\frac{1}{3}} + \left(1 + \frac{\Delta n}{n_{0e}}\right)^{\frac{1}{3}}}. \quad (104)$$

If magnetic field is small than $\Delta n/n_{0e} \ll 1$. In this regime we obtain

$$\omega_R^2 = \frac{1}{3} \frac{m_e}{m_i} k^2 v_{Fe}^2 \left(1 + \frac{1}{9} \frac{\Delta n^2}{n_{0e}^2} \right). \quad (105)$$

In the long-wavelength limit $\omega_{Ls} \gg kv_{Fs}$ the decrement of Landau damping for ion acoustic wave (103) can be rewritten as

$$\omega_{Im} = -\frac{\pi}{12} \frac{m_e}{m_i} kv_{Fe} \frac{1}{\left[\left(1 - \frac{\Delta n}{n_{0e}}\right)^{\frac{1}{3}} + \left(1 + \frac{\Delta n}{n_{0e}}\right)^{\frac{1}{3}}\right]^2}. \quad (106)$$

We can find simplification of formula (106) for regime of small magnetic field

$$\omega_{Im} = -\frac{\pi}{48} \frac{m_e}{m_i} kv_{Fe} \left(1 + \frac{2}{9} \frac{\Delta n^2}{n_{0e}^2} \right). \quad (107)$$

E. Zeroth sound

The zeroth sound is a well-known high frequency solution of the dispersion equation for degenerate ions. Usually one obtains it for an equal occupation of spin-up and spin-down states. Now we consider the zeroth sound in regime of high difference in occupation of spin-up and spin-down states by degenerate electrons. In this case $\omega \sim kv_{Fd} \gg kv_{Fu} \gg kv_{Fi}$.

The zero-sound (Ref on Silin) appears at $\omega \rightarrow kv_{Fd}$

$$\omega = kv_{Fd} + \delta\omega, \quad (108)$$

where $\delta\omega \ll kv_{Fd}$

In this regime dispersion equation takes the following form

$$1 = \frac{3}{2} \frac{\omega_{Ld}^2}{k^2 v_{Fd}^2} \left[\frac{\omega}{kv_{Fd}} \ln \left(\frac{2kv_{Fd}}{\delta\omega} \right) - 2 \right]. \quad (109)$$

Equation (109) gives the following solution

$$\omega_0 = kv_{Fd} + \delta\omega = kv_{Fd} \left[1 + 2 \exp \left(-2 - \frac{2k^2 v_{Fd}^2}{3 \omega_{Ld}^2} \right) \right]. \quad (110)$$

Let us present here the Fermi velocity of spin-down electrons via the conventional Fermi velocity

$$v_{Fd} = v_{Fe} \left(1 + \frac{\Delta n}{n_{0e}} \right)^{\frac{1}{3}}, \quad (111)$$

with $v_{fe} = (3\pi^2 n_{0e})^{\frac{1}{3}} \hbar / m$.

More explicit form of the zeroth sound spectrum (110) appears at substitution (111) in formula (110). Hence we have

$$\omega_0 = kv_{Fe} \left(1 + \frac{\Delta n}{n_{0e}} \right)^{\frac{1}{3}} \times \left[1 + 2 \exp \left(-2 - \frac{4k^2 v_{Fe}^2}{3 \omega_{Ld}^2} \left(1 + \frac{\Delta n}{n_{0e}} \right)^{-\frac{1}{3}} \right) \right]. \quad (112)$$

We can also consider regime of small difference in occupation numbers as well. In this case $kv_{Fd} \sim kv_{Fu}$, and $\omega \sim kv_{Fd} > kv_{Fu} \gg kv_{Fi}$.

$$\omega = kv_{Fd} + \delta\omega, \quad (113)$$

and

$$\omega - kv_{Fu} = \Delta + \delta\omega, \quad (114)$$

where $\Delta = k(v_{Fd} - v_{Fu})$.

In this regime the dispersion appears as follows

$$\omega = kv_{Fd} \left\{ 1 + 2 \exp \left[-2 - \frac{2k^2 v_{Fd}^2}{3 \omega_{Ld}^2} - \frac{n_{0u}^{\frac{1}{3}}}{n_{0d}^{\frac{1}{3}}} \left(2 + \ln \frac{n_{0d}^{\frac{1}{3}} - n_{0u}^{\frac{1}{3}}}{n_{0d}^{\frac{1}{3}} + n_{0u}^{\frac{1}{3}}} \right) \right] \right\}. \quad (115)$$

VII. CONCLUSIONS

Method of separate spin evolution quantum kinetics, which separately describes spin-up and spin-down electrons, has been developed. This method has been applied to rederivation of spectrum of the Langmuir waves and the SEAWs obtained earlier in terms of SSE-QHD. Regime of wave propagation parallel to the external magnetic field has been considered at calculations of spectrum of magnetised spin-1/2 quantum plasmas. Contribution of ions dynamics in dispersion of SEAW has been considered. Calculation of the Landau damping of the SEAW has been performed. Influence of separated spin evolution on real and imaginary parts of spectrums of ion-acoustic waves and zeroth sound have been found.

Calculation of the Landau damping of the SEAWs has demonstrated that the SEAWs are weakly damped waves. Thus we have shown that hydrodynamic calculations of the real part of spectrum of the SEAWs were reasonable.

We have presented fundamental applications of the separates spin evolution quantum kinetics method. Furthermore, this method, along with the developed earlier SSE-QHD, creates strong background for research of spin-1/2 quantum plasmas. It opens possibilities for more detailed analysis of various effects in quantum plasmas than usual spin-1/2 QHD or similar quantum kinetics.

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